ANALYSIS OF AN FDDI NETWORK SUPPORTING MULTIPLE-PRIORITY STATIONS WITH SINGLE-PACKET BUFFERS

IZHAK RUBIN and JAMES C.-H. WU
Electrical Engineering Department, 58-115 ENGR IV
University of California, Los Angeles, CA 90024-1594

ABSTRACT We present exact analysis of a non-symmetric multi-priority token ring network with single packet buffers under a timed-token protocol, similar to that employed for medium access control of the Fiber Distributed Data Interface (FDDI) network. An efficient iterative procedure is used to compute the limiting state distributions of the embedded Markov chains representing the system state process. We obtain the distributions of the token rotation time, the normalized throughput, and the mean packet waiting time. By using conditional residual delay analysis, moments of the packet delay are computed. We illustrate the application of the analytic approach through numerical examples representing FDDI network systems under various traffic conditions.

1 Introduction

Timed-token protocols (TTP) [1]-[2] have long been of practical interest in affecting the sharing of a medium by multiple service streams. Under the ANSI-based timed-token ring standard, FDDI [3]-[4], and the IEEE-defined timed-token bus standard, 802.4 [5], two classes of service are provided: 1) A synchronous class of service is intended for applications that require guaranteed bandwidth and response time, such as real-time packet voice/video or control signals; 2) An asynchronous class of service is available for multiple priority levels of traffic used for applications such as file transfer or bursty interactive data, that share the available system bandwidth dynamically.

Time-domain analyses of cyclic service systems with single buffers have been previously conducted for a moderate number of stations connected to the ring network. Tsai and Rubin [6] study the queuing behavior of symmetric token ring networks governed by the IEEE 802.5 token ring standard. Moments of message delay are obtained by solving a set of recurrence equations based on the regenerative property of the conditional residual delay. Takagi [7] studies the effects of the token rotation time on the delay-throughput performance of a symmetric single-buffer message-prioritized system operating under a timed-token protocol. The limiting state distribution of the embedded Markov chain is calculated by the Gauss elimination method. The waiting time distribution of a synchronous message is derived by representing it as the forward recurrence time distribution of the token rotation time. By using the mean waiting time of this message and the relationship between mean delay and throughput for a single-buffer station, the mean waiting time of an asynchronous message is obtained.

The inherent chaining property in a polling system is revealed when the system is examined during token visits to the stations. By exploiting this chaining property, Ferguson and Aminetash [8] present exact analyses of polling systems with exhaustive and gated services, while Konheim [9] provides an approximate analysis of a polling system with limited service. Note, however, that under the FDDI TTP, dynamic time limits are imposed on the token holding duration permitted at each station.

In this paper, we analyze a nonsymmetric multi-priority token ring network with stations which have single buffers, under a timed-token protocol similar to that employed by the FDDI medium access control. A station can provide access support to a traffic stream on a synchronous or asynchronous (of any priority level) basis. The analysis carried out in this paper extends the methods used in [6]-[7] to such an FDDI-type system. The system state process is characterized through a set of discrete-time Markov chains involving the state of the stations as observed at the instants of token arrival. An efficient iterative procedure is used to compute the limiting state distributions of the embedded Markov chains, without inverting the transition probability matrix. Moments of the packet delay are computed by using a conditional residual delay analysis. We investigate the performance of such FDDI network systems when loaded by various characteristic traffic configurations.

2 System description

We consider a nonsymmetric token ring network in which the medium is shared in accordance with a timed-token protocol such as the FDDI MAC scheme. The network consists of K stations (numbered as 0, 1, ..., K-1), each of which has a buffer that can accommodate only a single packet. Packets arrive at station-k according to a Poisson process with rate \( \lambda_k \) (packets/msec), \( k = 0, 1, \ldots, K - 1 \). The arriving packet at station-k is composed of a geometrically distributed number of fixed-size segments with parameter \( p_k \), so that the average number of segments contained in a packet is equal to \( p_k^{-1} \). The segment transmission time at station-k is equal to \( b_k \) (msec).

A token circulates around the ring and visits station-0, station-1, ..., station-(K-1) cyclically. The walk time (ring latency) for a complete circulation of the token around the ring (not including the packet transmission time) is equal to \( R \) (msec). A target token rotation time (TTRT) is selected during ring initialization. Multiple priority levels of asynchronous service are supported and distinguished by the use of distinct timing thresholds: \( T_{pri-i} \) for asynchronous priority-i service, \( i \geq 1 \), where \( T_{pri-i} = TTRT \), \( T_{pri-i} <

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3.1 The Embedded state processes

We examine the system at the instants of token arrivals at the stations. Let $r_{n,j}$ denote the instant of the n-th token arrival at station-k, $k = 0, 1, ..., K - 1$; $n \geq 0$. The state of station-k at time $r_{n,j}$ is defined as follows:

$s^k_n = 1$ if station-k has a packet queued and a segment will be transmitted,

$s^k_n = 2$ if station-k has a packet queued that cannot be transmitted.

Note that for a synchronous station, $s^k_n \neq 2$, since a synchronous station with a packet queued can always transmit a segment when the token arrives at the station.

We define the n-th token rotation cycle with respect to station-k to be the n-th token interarrival period at station-k, $[r_{n-1}^k, r_n^k)$, $n \geq 1$. The corresponding token rotation (cycle) time is denoted as $C^k_n$. Note that $C^k_n = r_n^k - r_{n-1}^k = R + \sum_{i=0}^{n-1} b_i I(s^k_{i-1}) + \sum_{i=0}^{k-1} b_i I(s^k_i) = 1$.

The states of the stations in the system during the n-th token rotation cycle with respect to station-k, $k = 0, 1, ..., K - 1$, are denoted by a $K$-tuple vector $S^k_n = (s^0_n, s^1_n, ..., s^K_n)$. For each $k \in \{0, 1, ..., K - 1\}$, the state process $(S^k_n)$ is a discrete-time Markov chain over the state space $A^k$.

$A^k = \{ \{u_0, u_1, ..., u_{K-1}\} | u_j = 0, 1, 2; j = 0, 1, ..., K - 1 \}$.

We note that a total of $K$ discrete-time Markov chains are defined: $(S^0_n), (S^1_n), ..., (S^{K-1}_n)$. The number of the overall states in $A$, denoted as $|A|$, is noted to be equal to $3^K$.

3.2 The balance equations

The limiting state distributions are defined as:

$\pi_k(S^k_n) = \lim_{n \to \infty} P(S^k_n = S_k) \quad S_k \in A; k = 0, 1, ..., K - 1,$

where $S_k = (s^0_k, ..., s^{K-1}_k)$.

For each $k \in \{0, 1, ..., K - 1\}$, the underlying Markov chain $(S^k_n)$ has a finite state space and is noted to be irreducible and aperiodic. Therefore a unique limiting (steady-state) distribution exists.

The steady-state probabilities satisfy the following set of balance equations:

$\pi_{k+1}(S_{k+1}) = \sum_{S_k \in A} \pi_k(S_k) P(S_{k+1} = S_{k+1} | S_k = S_k), \quad k = 0, 1, ..., K - 1$.

(3)

$S_{k+1} \in A; k = 0, 1, ..., K - 1$.

and

$\sum_{S_k \in A} \pi_k(S_k) = 1, \quad k = 0, 1, ..., K - 1.$

(4)

The following is noted concerning Eqs.(3)-(4).

- In writing Eq.(3), we employ the equivalent expression by defining $\pi_k(S_0, S_1, ..., S_{K-1}) = \pi_k(S_0, S_1, ..., S_{K-1})$ for notational simplicity.

6.5.2
the system, increases. We therefore present the following iteration procedure, which proceeds in a round-robin fashion, instead of matrix inversion, the transition matrix, to be given by

\[
\text{Procedure (iteration)} \quad v_k = \sum_{i=0}^{n-1} b_i I(s_i = 1).
\]

We observe that the state transition matrix is sparse due to the chaining property of the cyclic service system. The matrix representations in Table 1. Let \( N_j \) denote the number of transitions between states \( s_j \) and \( s_{j+1} \). We have \( N_j = 5 \cdot 3^{K-1} \). Let \( N_f \) denote the number of state transitions in a fully connected arrangement over state space \( A \). We note that \( N_f = |A|^2 = 3^{2K} \). This property of sparsity suggests that the limiting state distributions can be evaluated very effectively by iterative computations instead of matrix inversion, as \( K \), the number of stations in the system, increases. We therefore present the following iteration procedure, which proceeds in a round-robin fashion, for the calculation of the limiting state distributions.

**Procedure 1 Iterative Computations of \( \{\pi_k(s_k)\} \)**

1. **Initialization** \( i \leftarrow 0; \)
   \( \pi_0^{(i)}(s_0) = 1; \quad \pi_0^{(i)}(s_0) = 0, \quad \forall s_0 \in A, s_0 \neq 0. \)

2. **Iteration** (successive computations):

   \( \pi_0^{(i)}(s_0) \rightarrow \pi_1^{(i)}(s_1) \rightarrow \ldots \rightarrow \pi_{K-1}^{(i)}(s_{K-1}) \rightarrow \pi_0^{(i+1)}(s_0) \)

   - For \( k = 0, 1, \ldots, K - 2 \), calculate \( \pi_k^{[i+1]}(s_{k+1}) \), given \( \pi_k^{(i)}(s_k) \), according to Eq. (5).
   - \( i \leftarrow i + 1; \)

3. **Calculate** \( \pi_0^{(i)}(s_0) \), given \( \pi_{K-1}^{(i-1)}(s_{K-1}) \), according to Eq. (5).

3.3 Iterative calculations of the limiting state distributions

By examining the balance equations (see Eq. (3)), we observe that the state transition matrix is sparse due to the chaining property of the cyclic service system. The matrix represented by Table 1 is noted to have at most 5 positive entries. Let \( N_f \) denote the total number of transitions between states \( s_j \) and \( s_j \). We observe that \( N_f = 5 \cdot 3^{K-1} \). Let \( N_f \) denote the number of state transitions in a fully connected arrangement over state space \( A \). We note that \( N_f = |A|^2 = 3^{2K} \). We thus obtain the ratio of \( N_f \) to \( N_f \), representing the index of sparsity associated with the state transition matrix, to be given by \( \frac{N_f}{N_f} = 3^{2K} \). This property of sparsity suggests that the limiting state distributions can be evaluated very effectively by iterative computations instead of matrix inversion, as \( K \), the number of stations in the system, increases. We therefore present the following iteration procedure, which proceeds in a round-robin fashion, for the calculation of the limiting state distributions.

### Table 1: Evaluation of \( h_k^{[i]}(s_k) \)

<table>
<thead>
<tr>
<th>( s \rightarrow s_k )</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( e^{-\lambda_{V_k}} )</td>
<td>1</td>
<td>( 1 - e^{-\lambda_{V_k}} )</td>
</tr>
<tr>
<td>1</td>
<td>( p_k e^{-\lambda_{V_k}} )</td>
<td>0</td>
<td>( 1 - p_k e^{-\lambda_{V_k}} )</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>( I_{k}^{[i]}(s_k) )</td>
<td></td>
</tr>
</tbody>
</table>

### 3.4 Statistics for performance evaluation

Let \( C_k(t) \), \( t = t_0, t_1, \ldots, t_{\text{max}} \) denote the token rotation (cycle) time distribution at station-\( k \), where \( t_0, t_1, \ldots, t_{\text{max}} \) are the limiting state distributions.

\[
C_k(t) = \lim_{n \to \infty} P(C_k^n = R + t) = \sum_{t=0}^{t_{\text{max}}} \pi_t(s_k),
\]

where \( \pi_t(s_k) \) is noted to have at most 5 positive moments.

4 Analytic calculation of the moments of packet delay

In this section, we derive expressions for the first and second moments of packet delay. Moments of higher orders can be computed similarly. Let \( D_k \) denote the packet delay at station-\( k \). We observe that \( D_k \) can be decomposed into the following two components: the forward recurrence token intervisit time, denoted as \( V_k \), and the packet waiting time, denoted as \( D_k \). The forward recurrence token intervisit time at station-\( k \), \( V_k \), is measured from the instant that the packet arrives at station-\( k \) to the instant that the packet departs from station-\( k \). According to the model's assumptions, we have that \( G_k \in \{0, b_k\} \).

We then obtain the normalized throughput of station-\( k \) to be given by

\[
\rho_k = \frac{G_k}{C_k},
\]

where \( C_k \) is the mean token rotation time with respect to station-\( k \), computed as \( C_k = \sum t_i C_k(t) + R \).

The mean packet delay at station-\( k \), \( D_k \), can be computed from \( \rho_k \) as follows:

\[
D_k = \frac{b_k}{p_k \rho_k} - \frac{1}{\lambda_k}.
\]

The mean packet waiting time at station-\( k \), \( W_k \), is thus obtained by subtracting the mean packet transmission time, \( \frac{b_k}{ho_k} \), from the mean packet delay, \( D_k \):

\[
W_k = D_k - \frac{b_k}{p_k} = \frac{b_k}{p_k} - \frac{1}{\rho_k} - \frac{b_k}{p_k}.
\]
arrives at station-\( k \) to the instant that the packet departs from station-\( k \). We have \( D_k = V_k + \Delta_k \). It is observed that, conditioned on the states of the underlying Markov chain, the two delay components, \( V_k \) and \( \Delta_k \), are statistically independent. We therefore present the following computational methods for calculating the moments of packet delay.

### 4.1 The forward recurrence token intervisit time

The first two moments of the forward recurrence token intervisit time, conditioned on the states of the underlying Markov chain, are given as: (see \[7\])

\[
E(\hat{V}_k | \mathbf{s}_k) = \frac{V_k - \frac{1}{\lambda_k} (1 - e^{-\lambda_k V_k})}{(1 - e^{-\lambda_k V_k})} \tag{11}
\]

\[
E(\hat{V}_k^2 | \mathbf{s}_k) = \frac{V_k^2 - \frac{2}{\lambda_k} V_k + \frac{2}{\lambda_k^2} (1 - e^{-\lambda_k V_k})}{(1 - e^{-\lambda_k V_k})} \tag{12}
\]

### 4.2 The residual packet delay

By employing the regenerative property (with respect to cycle start times at station-\( k \)) of the residual packet delay, \( D_k \), conditioned on the states of the embedded Markov chain, the first two moments of the residual packet delay can be computed by solving the following set of simultaneous equations.

\[
\begin{align*}
E(\hat{D}_k | \mathbf{s}_k) &= \left\{ \begin{array}{l}
p_k b_k + (1 - p_k) \left[ C_k + \sum_{\mathbf{L} \in \mathcal{A}, \mathbf{s}_k \neq 0} E(\hat{D}_k | \mathbf{s}_k) \right], \\
C_k + \sum_{\mathbf{L} \in \mathcal{A}, \mathbf{s}_k \neq 0} E(\hat{D}_k | \mathbf{s}_k) & s_k = 1 \\
\hat{P}_k(\mathbf{s}_k' | \mathbf{s}_k), & s_k = 2
\end{array} \right. \tag{13}
\end{align*}
\]

\[
\begin{align*}
E(\hat{D}_k^2 | \mathbf{s}_k) &= \left\{ \begin{array}{l}
p_k b_k^2 + (1 - p_k) \left[ C_k^2 + \sum_{\mathbf{L} \in \mathcal{A}, \mathbf{s}_k \neq 0} [2C_k E(\hat{D}_k | \mathbf{s}_k) + E(\hat{D}_k^2 | \mathbf{s}_k)] \right], \\
C_k^2 + \sum_{\mathbf{L} \in \mathcal{A}, \mathbf{s}_k \neq 0} [2C_k E(\hat{D}_k | \mathbf{s}_k) + E(\hat{D}_k^2 | \mathbf{s}_k)] & s_k = 1 \\
E(\hat{D}_k^2 | \mathbf{s}_k) \hat{P}_k(\mathbf{s}_k' | \mathbf{s}_k), & s_k = 2
\end{array} \right. \tag{14}
\end{align*}
\]

where the transition probabilities, \( \hat{P}_k(\mathbf{s}_k' | \mathbf{s}_k) \), are defined as follows. Let \( \{ \mathbf{c}_k^n = 1 \} \) denote the event that the packet queued at station-\( k \) at time \( \mathbf{s}_k^n \) still resides in station-\( k \) immediately after the \( n \)-th token departure from station-\( k \). We have

\[
\hat{P}_k(\mathbf{s}_k' | \mathbf{s}_k) = \begin{cases} 
\mathbf{c}_k^n = 1 & \mathbf{c}_k^n = 1 \\
\mathbf{c}_k^n = 0 & \mathbf{c}_k^n \neq 1
\end{cases}
\]

\[
\hat{P}_k(\mathbf{s}_k' | \mathbf{s}_k) = \prod_{i=0}^{K-1} b_{i, s_k(i)} \prod_{i=0}^{K-1} b_{i, s_k(i)} \left( \sum_{j=0}^{K-1} b_{j, s_k(j)} \right) \left( \sum_{j=0}^{K-1} b_{j, s_k(j)} \right)
\]

\( \forall \mathbf{s}_k, \mathbf{s}_k' \in \mathcal{A}, \ s_k, \ s_k' \in \mathcal{A}, \ s_k = 1, 2; \ k = 0, 1, \ldots, K - 1. \tag{15} \)

### 4.3 Moments of the packet delay

We obtain moments of the packet delay by unconditioning the above-mentioned moments of the conditional delay components:

\[
\begin{align*}
\hat{D}_k &= \sum_{\mathbf{L} \in \mathcal{A}, \mathbf{s}_k \neq 0} \left[ E(\hat{V}_k | \mathbf{s}_k) + \sum_{\mathbf{L} \in \mathcal{A}, \mathbf{s}_k \neq 0} E(\hat{D}_k | \mathbf{s}_k) \hat{P}_k(\mathbf{s}_k' | \mathbf{s}_k) \right] \hat{s}_k(\mathbf{s}_k) \tag{16}
\end{align*}
\]

\[
\begin{align*}
E(\hat{D}_k^2) &= \sum_{\mathbf{L} \in \mathcal{A}, \mathbf{s}_k \neq 0} \left[ E(\hat{V}_k^2 | \mathbf{s}_k) + \sum_{\mathbf{L} \in \mathcal{A}, \mathbf{s}_k \neq 0} [2E(\hat{V}_k | \mathbf{s}_k) E(\hat{D}_k | \mathbf{s}_k) + E(\hat{D}_k^2 | \mathbf{s}_k)] \right] \hat{s}_k(\mathbf{s}_k) \tag{17}
\end{align*}
\]

where

\[
\hat{s}_k(\mathbf{s}_k) = \left\{ \begin{array}{l}
(1 - e^{-\lambda_k V_k}) \hat{s}_k'(\mathbf{s}_k), \quad \mathbf{s}_k = 0 \\
\frac{1}{p_k} \hat{s}_k(\mathbf{s}_k), \quad \mathbf{s}_k = 1 \end{array} \right. \tag{18}
\]

\[
\hat{s}_k(\mathbf{s}_k) = \left\{ \begin{array}{l}
\hat{s}_k(\mathbf{s}_k), \quad \mathbf{s}_k = 0 \\
\hat{s}_k(\mathbf{s}_k), \quad \mathbf{s}_k = 1 \end{array} \right. \tag{19}
\]

## 5 Illustrative applications

We illustrate the application of the analytic approach developed above by considering the following examples.

First, we consider a symmetric token ring system with 8 stations. The walk time of the system is equal to 0.32 msec (corresponding to a geographic span of 60 km). We set \( p_k = 1 \), so that a packet consists of a single segment. The transmission time of a packet (or a segment), \( b_k \), is set equal to 0.32 msec (corresponding to the transmission of a packet containing 4 Kbytes). We observe the queuing behavior of the system at various loading conditions. For the results shown in Fig.1(a), the stations are all provided asynchronous priority-1 service, and we set \( T_{pri-1} = T_{RTT} = 2.24 \) msec, corresponding effectively to the time elapsed in the transmissions of 6 segments. We note that the token rotation time distributions follow approximately the following distribution functions: exponential (for system throughput \( \rho = 0.08 - 0.68 \), uniform (\( \rho = 0.72 \), and reversed exponential (\( \rho \geq 0.79 \)). In Fig.1(b)-(c), we illustrate the mean packet waiting time and system throughput under various timing threshold levels. As expected, the system throughput increases and the mean waiting time decreases as the \( (T_{pri}) \) timing threshold is increased. The symmetric loading character of this system configuration does not require the selection of station timing thresholds to limit dwell times to guarantee response time for certain hyperactive stations.

We next consider a token ring system with 10 stations. The stations around the ring are alternatively provided with asynchronous priority-1 and priority-2 services. Symmetric station loading is assumed. However, the asynchronous priority thresholds are set to be: \( T_{pri-1} = 2.24 \) msec, and \( T_{pri-2} = 1.6 \) msec (corresponding effectively to the transmissions of 6 and 4 segments, respectively). The walk time
and the segment transmission time are the same as those given in the previous example. From Fig. 2(a), we observe the effect of the two timing thresholds on the token rotation time distributions. In Figs. 2(b)-(c), we observe variation of the queueing performance for the two classes of stations. The mean waiting time of priority-2 packets increases significantly and the throughput of priority-2 traffic diminishes as the mean token rotation time exceeds the timing threshold $T_{pri-2}$.

We next consider the example of a multi-priority system with 5 stations, each of which is associated with a single asynchronous priority level of service. The stations are symmetrically loaded with $p_k = 0.9$, $b_k = 1$ msec. The timing thresholds are selected to be $T_{pri-1} = 5.2$, $T_{pri-2} = 4.2$, $T_{pri-3} = 3.2$, $T_{pri-4} = 2.2$, $T_{pri-5} = 1.2$ msec. The walk time is set equal to 0.2 msec. In Fig. 3(a), we show the delay vs. throughput performance curves for each station. Stations with higher priority level of service experience lower packet delay and are provided with higher throughput as the offered load of the system is increased. In Fig. 3(b), we show the coefficient of variation of packet delay (expressing the ratio of the standard deviation to the mean of the packet delay), defined by $\frac{\sigma^D}{D}$ for each station. We note the higher priority station experiences lower packet delay variations. As the network loading increases, packets at lower priority stations incur more often longer delays (since their transmissions are deferred to future token visits).

References