Abstract

We study the queueing performance of source stations and network switches when an input rate flow control mechanism is applied to regulate the traffic flow between source stations and network switches. We demonstrate the ensuing tradeoffs between message delays imposed at the source stations by the use of the flow regulation mechanism and the message delays incurred by messages at the internal switches as effected by the statistical features of the incoming message streams. We first examine the increased queue size and message delays incurred at source stations by the employed input rate flow control access mechanism. The statistical behavior of the output traffic from source stations is then characterized. The impact of input rate flow control on message delays incurred at the internal network switches are then analyzed. Performance curves are presented to illustrate the queueing behavior and message delays at source stations and at the internal network switches. Using our results, the system designer can properly select the parameters of the flow control scheme to guarantee acceptable limits of queue-sized and message delays at the source stations and at the network switch.

1. Introduction

Input rate flow control is an effective mechanism for the regulation of traffic flow into high-speed communication networks. Traditional window flow control schemes alone are insufficient as the speed of transmission increases, due to the long associated end-to-end delays incurred. BELLCORE's Switched Multimegabit Data Service (SMDS) [1] and IBM's Packetized Automatic Routing Interated System (PARIS) [2] are two examples of high speed networks which implement input rate flow control mechanism. ATM (Asynchronous Transfer Mode) networks which provide support for B-ISDN services also tend to use input rate control schemes.

Our model for an input rate flow control mechanism is depicted in Figure 1. It consists of a cell queue and a credit (token) queue. The credit queue has a maximum buffer size denoted as \( C_{\text{max}} \). Cell queue is assumed to have an infinite buffer size. Credit is generated at a fixed rate and the unused credit is stored in the credit queue. Cells are generated by end user stations. Every cell requires a single credit in order for it to be transmitted into the network. When insufficient credit is available, cells are stored in the cell queue.

Early analysis of the queueing performance of the source stations under input rate flow control mechanism has been carried out in [3][4][5]. In this paper, we extend our previous results[3] to analyze the queueing performance at the backbone network.

Our models are presented in section 2. In section 3, numerical examples and performance curves are provided to illustrate the methodology developed in this paper. Conclusion are drawn in section 5.

2. Queueing Models

Our queueing models are depicted in Figure 2. There are two stages of queueing systems; the first stage represents the source stations and the second stage involves the network switch. A source station is modeled as a multi-server queueing system with servers controlled by an input rate flow control mechanism, when such a control is applied to the source station. Messages are described as fixed-length ATM cells. Messages arrive in accordance with a geometric batch stochastic point process. Source stations are assumed to be stochastically independent of each other.

The output traffic from each source station is characterized as a Markov Modulated Process (MMP). These output streams are then fed into a network switch, which is modeled as an \( L \) server queueing system. The impact of the input rate flow control mechanism on the message delay performance at the network switch is illustrated as we compare the message delay performance at the network switch fed by these regulated (or unregulated) streams (i.e., streams smoothed by the input rate flow control and streams un-smoothed by the input rate flow control) with the message delays incurred at the source stations, under various parameter levels of the flow control schemes.

2.1 Source Stations Under Input Rate Flow Control

A slotted channel is presented to analyze the queueing behavior at the source stations when an input rate flow control mechanism is used. Time is segmented into fixed-length slots. Each slot duration is equal to the transmission time of a cell. The input rate flow control regulator is keeping track of the available credit the the station's interface with the switch. A frame consists of a consecutive of \( K \) slots. The credit is incremented by one every \( K \) slots, at the start of each frame. The following variables are then defined.

- \( X_n \): System size (number of cells queued in the source station's buffer) at the start of frame \( n \); \( X_n = 0, 1, 2, \ldots \)
- \( C_n \): Credit available at the start of frame \( n \); \( C_n = 1, 2, \ldots, C_{\text{max}} \)
- \( A_n \): Number of arriving cells during frame \( n \), as recorded at the end of frame \( n \); \( A_n = 0, 1, 2, \ldots \)
- \( D_n \): Number of departing cells from the source station during frame \( n \); \( D_n = 0, 1, 2, \ldots, K \)

Cells arriving within a frame are considered for admissions at the start of the next frame. Denote \( f(\text{flags}) \) to be an indicator function such that it equals 1 when all its flags are true and 0 otherwise. A set of recursive equations are written to describe...
the operation of the input rate flow control, as presented in the following.

\[ X_{n+1} = X_n + A_n - D_n \quad (2.1) \]

\[ C_{n+1} = \min \{ C_{\text{max}}, C_n - D_n + 1 \} \quad (2.2) \]

where \( D_n = \min \{ X_n, C_n, K \} \).

Thus, \( \{(X_n, C_n), n \geq 0\} \) is a discrete time Markov Chain and its steady-state distribution can be obtained as follows.

Define \( \Pi_{x,c} = \lim_{n \to \infty} P\{X_n = x, C_n = c\} \) and \( a_i = P\{A_n = i\} \).

We have the following equilibrium balance equations.

\[ \Pi_{x,c} = \sum_{j=0}^{K} \sum_{i=0}^{C_{\text{max}}} a_i \Pi_{x+j,c+i} \quad (2.4) \]

\[ \Pi_{x,c} = \sum_{i=0}^{K} a_i \Pi_{x+c-1,i} + \sum_{i=0}^{A_{\text{max}}} a_i \Pi_{x+c-1,i+1} \quad (2.5) \]

\[ \Pi_{x,C_{\text{max}}} = a_0 (\Pi_{1, C_{\text{max}}} + \Pi_{0, C_{\text{max}}}), \theta \]

\[ x = 0, 1, 2, \ldots \]

This set of equations can be solved numerically under the boundary condition:

\[ \sum_{x=0}^{X_{\text{max}}} \sum_{c=1}^{C_{\text{max}}} \Pi_{x,c} = 1. \quad (2.7) \]

The exact solutions for the joint distributions of \( \Pi_{x,c} \) can be obtained and the moments of the system size can be computed.

2.2 Output Traffic Characterization of Source Stations

Output traffic from a station is characterized as a Markov Modulated Process (MMP). The output traffic from a source station without input rate flow control can be shown to be a MMP with the burst position index, \( B_n \) (defined later in this section), as the modulating variable. The output traffic of a station with input rate flow control is also a MMP with \( (X_n, C_n) \) serving as the joint modulating variables. However, when the network switch is loaded by a number of source stations, the dimensionality of the state space can increase considerably under the use of this process to compute the performance of the second stage queueing system. Therefore, we approximate the output traffic from source stations regulated by input rate flow control by a MMP process which is defined similarly to the one characterizing the output traffic from source stations which are not regulated. The state space is then greatly reduced and the performance of the second stage queueing system can be readily and efficiently computed.

2.2.1 Output Traffic Unregulated by Input Rate Flow Control

When an input rate flow control mechanism is not used to regulate the source stations, source stations can burst into the network whenever they have cells waiting for transmission. Let \( D_n \) be the number of departures from source station during frame \( n \). We have:

\[ D_n = \min \{ X_n, K \}. \quad (2.8) \]

The burst position index, \( B_n \), is defined as follows:

\[ B_n = \begin{cases} B_{n-1} + 1, & \text{if } D_{n-1} = K \\ 0, & \text{otherwise} \end{cases} \quad (2.9) \]

The variable \( B_n \) is referred to as the mode of the MMP. One readily notes that \( \{B_n, n \geq 0\} \) is a Markov Chain. The transition probability function for \( B_n \), \( P\{B_{n+1}|B_n\} \), can be obtained and the conditional probability function of \( D_n \) given \( B_n \) and \( B_{n+1} \), \( P\{D_n|B_n, B_{n+1}\} \), can also be calculated. Please refer to [6] for detail. The following outlines the procedure used to obtain these probability functions. For each \( b \), we proceed as follows:

1. Find \( P\{X_n = x|B_n = b\} \).
2. Find \( P\{D_n = d|B_n = b\} \) from 1.
3. Find \( P\{B_{n+1} = b|B_n = b\} \) from 2.
4. Find \( P\{X_n = x|B_n = b\} \) from 1 and 3.

The above steps are used to iteratively compute the desired functions. The computations of the functions for \( B = b + 1 \) require only those obtained for \( B = b \), further contributing to the numerical efficiency of the calculations.

2.2.2 Output Traffic Under Input Rate Control Regulation

When an input rate flow control mechanism is used to regulate the source stations, source stations can send cells into the network only when sufficient credit is available for those cells. Let \( D_n \) be the number of departures from source station during frame \( n \). We have:

\[ D_n = \min \{ X_n, C_n, K \}. \quad (2.10) \]

This is the same as equation (2.3). From section 2.1 we recall that the process \( \{(X_n, C_n), n \geq 0\} \) is Markovian. We could model the output traffic as a MMP with \( (X_n, C_n) \) as the joint modulating variables. However, the state space would then become too large for practical computations when we need to super-impose the output traffic from a number of source stations subject to input rate flow control regulation and feed the regulated traffic into the network switch. Rather, we introduce the following approximation, where we model the station’s output process as a MMP modulated by an extended variable \( B_n \). We expand the definition of the burst position index, \( B_n \), to accommodate the extra mode needed to model the output traffic when it is regulated by an input rate flow control mechanism. The definition of \( B_n \) is set as:

\[ B_n = \begin{cases} B_{n-1} + 1, & D_{n-1} = K \text{ and } X_{n-1} > C_{n-1} \\ -1, & D_{n-1} < K \text{ and } X_{n-1} > C_{n-1} \\ 0, & \text{otherwise} \end{cases} \quad (2.11) \]

Figure 3 depicts the transition diagram of \( B_n \). Thus, the extended variable \( B_n \) involves an explicit state \((-1)\) which identifies a frame during which the system’s queue size is higher than the available credit, while only a portion of the frame is used for cell transmissions. This state \((-1)\) depicts an event that the source station’s transmissions are restricted which can frequently occur. We proceed to compute the transition probability function of \( B_n \), \( P\{B_{n+1}|B_n\} \), and the conditional probability of the number of departing cells given the current and the next modes, \( P\{D_n|B_n, B_{n+1}\} \). Defining \( B_{\text{max}} = \left\lfloor \frac{X_{\text{max}}}{C-1} \right\rfloor \), we present the calculation of those functions in the following three cases, based upon the value of \( B_n = b \).

I. \( B_n = b_1 = 0 \): In this case, we know \( 0 \leq D_{n-1} < K \) and \( X_{n-1} \leq C_{n-1} \). This is the case since otherwise the current mode, \( B_n \), wouldn’t be equal to 0. The following two identities are used:

\[ P\{D_n|B_n, B_{n+1}\} = P\{D_n|B_n, B_{n+1}\} \quad (2.12) \]

\[ P\{B_n, B_{n+1}\} = \sum_{d=0}^{K} P\{D_n = d, B_n, B_{n+1}\}. \quad (2.13) \]

We obtain the joint probabilities of \( \{D_n, B_n, B_{n+1}\} \) as follows.

\[ D_n = 0: \]

\[ P\{D_n = 0, B_n = 0, B_{n+1} = 0\} = a_0 \sum_{i=0}^{K} \sum_{j=i}^{C_{\text{max}}(i,j)} P\{X_{n-1} = i, C_{n-1} = j\}. \quad (2.14) \]

\[ D_n = 1: \]

\[ P\{D_n = 1, B_n = 0, B_{n+1} = 0\} = \text{I.} \]
11. We then get
\[ \sum_{i=0}^{\min\{C_{\text{max}},K-1\}} P(X_n = i, C_{n-1} = i) = 1 \]
\[ P(D_n = 1, B_n = 0, B_{n+1} = -1) = P(X_n = C_{n-1}, X_{n-1} < K, A_{n-1} > 1) \]
\[ = \left\{ 1 - a_0 \right\} \sum_{i=0}^{C_{\text{max}}} P(X_{n-1} = i, C_{n-1} = i) \]
\[ + \left\{ 1 - a_0 \right\} \sum_{i=0}^{C_{\text{max}}} P(X_{n-1} = i, C_{n-1} = i) = 1 \]

\[ P(B_{n+1} = b | B_n = -1) = \left\{ \begin{array}{ll}
 0 & b = 0 \\
 1 & \text{otherwise}
\end{array} \right. \] (2.25)

11.1.3

III. \( B_n = 1, 2, \ldots, \text{Max} \) : We observe that to be in mode \( b \) at time \( n \), the following has to be hold:
\[ C_{n-1} \geq K + (b-1)(K-1) \text{ or } C(0) \]. (2.27)

We derive the conditional probability function of \( D_n \) and the transition probability function of \( B_n \) by the following two identities:
\[ P(D_n = d | B_n = b, B_{n+1} = b_d) = \frac{P(D_n = d | B_n = b, B_{n+1} = b_d)P(B_{n+1} = b_d)}{P(B_n = b)P(B_{n+1} = b_d)} \] (2.28)
\[ P(B_{n+1} = b_d | B_n = b) = \sum_d P(B_{n+1} = b_d | B_n = b) \] (2.29)

The following relationship is observed. We note that \( P(d) \) is the probability of having \( d \) departures in mode \( b \) when no regulation is imposed, as defined in section 2.2.1.

11.1.3.1 \( D_n = 0, B_{n+1} = 0 | B_n = b \) : \( P(D_n = 0, B_{n+1} = 0 | B_n = b) = P_0(b) \). (2.30)

11.1.3.2 \( D_n = 1, 2, \ldots, K-1 : \)
\[ P(D_n = d, B_{n+1} = 0 | B_n = b) = \left\{ \begin{array}{ll}
 1 & d = 0 \\
 \frac{d}{(b-1)K} & \text{otherwise}
\end{array} \right. \] (2.31)

From equations (2.12) to (2.20), we obtain \( P(D_n = d | B_n = b, B_{n+1} = b) \) for \( d = 0, 1, \ldots, \text{Max} \) and \( b = 0, 1, \ldots, \text{Max} \). We then get \( P(B_{n+1} = b | B_n = b) \) according to:
\[ P(B_{n+1} = b | B_n = b) = \sum_d P(D_n = d, B_n = b, B_{n+1} = b) \]
\[ = \sum_d \sum_j P(D_n = d, B_n = b, B_{n+1} = j) \] (2.21)
\[ = \left\{ \begin{array}{ll}
 1 & d = 0, b = 0, 1 \\
 0 & \text{otherwise}
\end{array} \right. \] (2.22)

11.1.3.1 \( D_n = K : \)
\[ P(D_n = d, B_{n+1} = 0 | B_n = b) = B_0(d) \] (2.32)

From equations (2.12) to (2.20), we obtain \( P(D_n = d | B_n = b, B_{n+1} = b) \) for \( d = 0, 1, \ldots, \text{Max} \) and \( b = 0, 1, \ldots, \text{Max} \). We then get \( P(B_{n+1} = b | B_n = b) \) according to:
\[ P(B_{n+1} = b | B_n = b) = \sum_d P(D_n = d, B_n = b, B_{n+1} = b) \]
\[ = \sum_d \sum_j P(D_n = d, B_n = b, B_{n+1} = j) \] (2.21)
\[ = \left\{ \begin{array}{ll}
 1 & d = 0, b = 0, 1 \\
 0 & \text{otherwise}
\end{array} \right. \] (2.22)

II. \( B_n = b_1 = -1 \) : In this mode, we observe the following:
\[ P(D_n = d | B_n = b, B_{n+1} = b) = \left\{ \begin{array}{ll}
 1 & d = b_1, b = 0 \text{ or } -1 \\
 0 & \text{otherwise}
\end{array} \right. \] (2.23)

The transition probability function of \( B_n \) is obtained by the following equations:
\[ P(B_{n+1} = b | B_n = -1) = \sum_d P(D_n = d, B_n = -1, B_{n+1} = b) \] (2.24)
\[ P(D_n = -1) = \sum_d P(D_n = -1, B_{n+1} = b) \] (2.25)

The joint probabilities of \( P(B_{n+1} = b, B_n = -1) \) is given by the following relationships:
\[ P(B_{n+1} = b, B_n = -1) = \left\{ \begin{array}{ll}
 a_0P(X_n = 1, C_n = 1), & b = 0 \\
 P(X_n > 1, C_n = 1), & b = 1 \\
 0, & \text{otherwise}
\end{array} \right. \] (2.26)

2.3 Queueing Models for the Network Switch

The network switch is modeled as a \( L \) server queueing system. It is driven by \( M \) station. The service time process is deterministic since cells are of fixed size. The arrival process is a superposition of \( M \) Markov Modulated Processes (MMP) obtained from the previous sub-section.

The framing structure used in the models for the source station is applied again here. For the network switch, the server can serve up to \( L \) cells in each frame. Denote \( Y_n \) as the system...
The derivation of our queueing models is completed. We have used the Seidel method under a properly truncated state network switch. In the second example at the network switch. In the first example the average system sizes at the source stations and terms of the the average system sizes at the source stations and evolution of rate flow control scheme. The outputs of the stations drive the local source station subject to an input rate control regulation i.e.:

$$X = \sum_{m=1}^{M} \left( B_{A,m} \right)$$

where

$$am = \begin{cases} p, & m = A \\ 1 - p, & m = 0 \\ 0, & \text{otherwise.} \end{cases} \quad (3.1)$$

where $A$ is the batch size (measured in cells). The input rate, $\lambda$ [cells/frame], is thus:

$$\lambda = \sum_{m} ma_m = Ap. \quad (3.2)$$

In the following, we discuss each model in detail.

### 3.1 Example 1: Two Asymmetrical Source Stations

In this example (Figure 4), we demonstrate the effects of regulating a source station which generates highly bursty traffic streams. The arrival rates of both stations are 0.5 [cells/frame], but station 2 generates a more bursty traffic stream, for which $A = 22$. Station 1 generates less bursty traffic streams, for which the size of an arrival batch is only $A = 8$. The credit is incremented by one unit every 5 slots, $K = 5$, for both stations. The service rate of the network switch is set as 2 [cells/frame].

We set $C_{max} = 8$ at station 1 and plot the performance of the system as a function of the $C_{max}$ at station 2 (Figure 6). The variable $X1$ denotes the average system size at the buffer of station 1 and is not affected by varying the value of $C_{max}$. The variable $X2$ represents the average system size at station 2. It decreases as we increase the value of $C_{max}$, as shown in Figure 6. $SW(A)$ is the average system size at the input queue to the switch. It is computed by using the approximation method described above. Its value increases when the value of $C_{max}$ is increased. By increasing $C_{max}$, we apply less regulation of station 2 traffic leading to the decrease of the queue size $X2$; however, as a result, the output process from station 2 is more bursty, causing higher queue-size at the switch. We also plot the average system size at the switch’s queue as obtained by simulation and denoted as $SW(S)$. We conclude that our analytical approximation is highly accurate.

The sum of the three average system sizes (at the source stations and at the network switch, $X1 + X2 + SW(A)$), represents the average total delay experienced by a randomly selected cell (noting that $A_1 + A_2 = 1$ in this example). This is a direct result from Little’s theorem. As shown in the figure, the average delay of a randomly selected cell decreases monotonically as $C_{max}$ is increased. This is expected, since we cannot do any better in terms of the average total delay by imposing an input rate flow control to regulate traffic. However, we do have to use input rate flow control to protect network buffering resources and as a measure of fairness control, limiting interference with (and unacceptable delay levels) for already admitted message streams.

### 3.2 Example 2: Local and Inner-Network Traffic

In this example (Figure 5), the network switch is fed by a local station, which generates highly bursty traffic streams, and an inner network traffic which is less bursty. The transmission rate of the network link is $G$ [cells/frame]. We model the network traffic as a truncated Poisson point process with rate $\lambda$ [cells/frame].

Performance results are plotted in Figure 7. As shown in the figure, the average system size at the switch, $SW(A)$, is reduced by choosing a lower $C_{max}$ value at the local station. However, this results with the increase of the average system size at the local station. It is of interest to use the input rate flow control scheme to reduce the burstiness of the traffic to protect the inner-network traffic. The variable $SW(S)$ represents the average system size at the network switch, when no input rate control is used to regulate the local traffic. The simulation results of the average system size at the network switch, $SW(S)$, are shown well. Again, we note our analytical approximation technique yield very accurate results.

### 4. Conclusions

In this paper, we present an analytical methodology for the analysis and performance evaluation of high-speed sub-networks when an input rate flow control mechanism is used to regulate the traffic generated by a number of stations directly connected.
to a network switch. Performance results are plotted to illustrate the use of our techniques and to demonstrate key system design trade-offs. Those results can be used to determine the parameters of the system and of the input rate flow control scheme, under prescribed system performance indices.

We have demonstrated the key factors affecting the performance tradeoffs concerning the queue-size and delay levels at the source stations vs. the resulting buffer occupancy and message delays at the shared fast packet switch. As the burst regulation ($C_{\text{max}}$ level) of the input rate control scheme is relaxed, the queue sizes at the source stations are decreased, while those at the switch buffer increase. Our performance analysis techniques provide the system designer with a tool to properly select the acceptable levels of the source and switch delay and buffer-size components.

5. References